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Computational Analysis of Punching Shear Models of Steel Fiber Reinforced Concrete Slabs

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KEYWORDS

Punching shear, steel fiber, slabs.

ABSTRACT

A computational analysis is presented to predict the ultimate and cracking shear strength of steel fiber reinforced concrete slabs. Different models are suggested considering the effect of concrete compressive and tensile strength, amount of flexural reinforcements, yield strength of the reinforcement bars and steel fiber properties (volume percent, aspect ratio, and type of steel fibers). The predicted results are compared with the experimental data found in literature and found good agreement.

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1. Introduction

The Flat plate slabs defined as a structural member which carried directly by the columns without beams or girders. Such type of structure has more space in addition to its pleasant appearance. Flat plates are also economical in their framework, which represents a great part of the cost of reinforced concrete slabs.

Punching shear failure takes place when a plug of concrete is pushed out from the slabs immediately above out of the cone or pyramid cross-section at least as large as the loaded area [1]. Punching shear failure of slabs is usually sudden and leads to progressive failure of flat plate structures, therefore, caution is needed in the design of the slabs and attention should be given to avoid the sudden and failure condition. Random distribution of steel fibers to conventional concrete offers a convenient and practical means of achieving improvement in many of the engineering properties of the concrete such as tensile strength & compressive strength [2]. Many other benefits arise from using steel fibers such as shear reinforcement in flat plates instead of the conventional shear reinforcement [3].

Different types of models or equations are proposed based on linear and nonlinear regression analysis to study the effect of steel fibers, flexural reinforcement and concrete properties on the ultimate and cracking shear strength of steel fiber reinforced concrete slabs.

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2. Review of Literature

Yitiaki [4] presented a correlation between the punching resistance and flexural strength of slabs, he showed that the punching shear strength depends mainly on the yield strength of reinforcement and compressive strengthened concrete. Herzog [5] derived a simple empirical formula to estimate the punching shear of slabs. Al Ani [6] studied the effect of steel fiber on punching shear strength of conventionally reinforced concrete slabs. The author studied the effects of type, volume, fraction, and location of steel fibers on punching shear strength.

Oukaili and Salman [7] presented an experimental study on punching shear of reinforced concrete slabs with openings in the vicinity of the column. The authors concluded that all specimens are failed in punching shear. In addition, the existence of opening decreases the punching shear capacity and stiffness of slabs with respect to the control solid slabs and depending on the size and location of these openings with respect to the column.

Al-Mamoori [8] presented an experimental investigation on punching shear strength and failure of self-compacting concrete slabs using CFRP bars as internal strengthening in the slab-column connection region. The author showed that using high tensile CFRP bars improve the bearing capacity of reinforced concrete two-way slabs, and the effectiveness of the CFRP bars depends on the distribution and arrangement in the slab-column region.

Sarsam and Hassan [9] presented an experimental study on punching shear of flat slabs with steel fiber using reactive and modified powder concrete. The authors studied the effect of steel fiber and absence of coarse aggregate on compressive strength, splitting tensile strength, modulus of rupture and modulus of elasticity of concrete. Moreover, these authors studied the effect of steel fiber, steel reinforcement ratio and slab thickness on the failure characteristics of punching shear of slabs. Obtained results suggested that the existence of steel fiber enhanced the stiffness of slabs and reducing crack width and crack propagation, also decreasing the perimeter of the punching shear area.

Ju et al. [10] developed the punching shear model for steel fiber reinforced concrete taking into account the shear contribution of the concrete in the compression zone and that of the steel fiber at the crack interface. The proposed model considered the depth of the neutral axis, the crack angle, the shear contribution of compression concrete zone, steel fiber and critical perimeter of the concrete section. The proposed model showed an accurate prediction of the punching shear strength of the specimens found in the literature.

Mu and Meyer [11] presented an experimental study on the effect of the fiber-reinforced glass aggregate on punching shear resistance of slabs. The specimens were reinforced either by randomly distributed short fiber or by continuous fiber mesh with an equal fiber volume ratio. Obtained results showed that that fiber mesh is more effective in bending while the randomly distributed fibers are more effective in punching shear.

Hanai and Holanda [12] presented a study on the similarities between punching shear in slabs and shear strength of beams. The results showed that there are inequivalent similarities between them. The analogous slabs and beams should have some height, longitudinal reinforcement and concrete properties. The shear tests on small prismatic beams can be performed to get useful indicators for the steel fiber reinforced concrete mixture design, and therefore used for flat-slab column connections. In addition, these similarities can give information about the ductility of the connections.

Jatule and Karlurkar [13] investigated the punching shear behavior of high strength steel fiber reinforced concrete simply supported square slabs. The authors studied the effect of span to depth ratio, volume fraction of steel fiber, slab thickness, concrete strength and size of load-bearing plate. The results indicate that the increasing of steel fiber content or thickness of the slab leads to increasing in punching shear strength and ductility of the slabs. Further, the authors compared the simulated results of the ultimate punching shear using different code equations with experimental data found in literature and showed good agreement.

Al-shaikli [14] studied the behavior and punching shear of square and triangular slabs using reactive powder concrete to produce ultra-high-strength concrete. The results indicate that using steel fibers content (0.5% and 1%) increase the punching shear about (37% & 100%) respectively for square slabs, while about (53 and 100%) for triangular slabs.

Choi et al. [15] developed a new strength model for predicting punching shear of interior slab-column connections made of steel fiber reinforced concrete. The proposed equation is verified using existing data from literature and found very good accuracy.

Maya et al. [16] presented a mechanical model for predicting the punching shear strength of concrete steel fiber reinforced concrete slabs. The proposed model is verified with a wide number of experimental data and showed good accuracy.

Rajab [17]; studied the behavior and punching shear strength of slabs made from steel-reinforced self-compacting concrete using a nonlinear Finite Element analysis. Estimated results compared with the experimental results and showed good agreement. Moreover, the author showed that the addition of (0.75%) of steel fiber increases the punching shear by about (15%).

Cheng and Montesiens [18]; presented and experimental study on punching shear of steel fiber reinforced concrete slabs, using different types of fibers (hooked and twisted), fiber strength, fiber volume fraction and reinforcement ratio of the slab. The authors concluded that the best behavior of shear strength – rotation interaction is achieved by using (1.5%) volume fraction of steel fiber. Additionally, punching shear increased by about (55%) and rotation capacity by about (125%). The failure mode changed from punching shear failure to flexural yielding failure.

Magdum and Veerabhadrannavap [19] presented an analytical analysis using the Finite Element method by ANSYS for conventionally reinforced concrete slab-column joints with steel fiber. They concluded that steel fiber reinforced concrete slabs showed (15%) less deformation compared to conventionally reinforced slabs subjected to axial load on the column face and reaction force at the bottom face, while (24%) less during the axial load, gravity load. Further, the authors found the steel fiber reinforced concrete slabs tend to deform (50%) less compared to conventional reinforced specimens.

Higashiyama et al. [20] proposed a design equation for the punching shear capacity of steel fiber reinforced concrete based on Japan society of civil Engineer's standard specifications. Accordingly, the authors concluded that the addition of steel fiber improves mechanical behavior, ductility, and fatigue strength of concrete. These authors have also studied the effect of steel fiber properties (volume fraction and type), slab thickness, steel reinforcement ratio and compressive strength, and then the predicted punching shear compared with those found in literature in which a good accuracy was found.

Minh et al. [21] studied the behavior and capacity of steel fiber reinforced concrete flat slabs under punching shear force. The experimental results show that the punching shear capacity and improvement of cracking behavior are increased with the addition of steel fibers. In addition, ductility of the slabs is increased and crack width decreased by about (70.8%) with the addition of steel fibers.

Mondo [22] proposed a method for predicting the residual tensile strength and shear strength based on fiber content properties, cylindrical compressive strength of concrete and amount of the flexural reinforcement using a wide range of experimental data found in the literature. The calculated results showed to be satisfactory when compared with the experimental results.

Al-Quraishi [23] concluded that the slab without steel fiber failed by punching shear, while the slab with steel fiber failed by splitting of concrete cover. The effect of steel fiber content, compressive strength, tension reinforcement ratio, size effect and yield strength of tension reinforcement are considered to propose a numerical model using finite element analysis for ultra-high performance concrete slabs. The proposed design equation of UHPC slabs is modified to include HSC and NSC slabs without steel fiber and with steel fiber. Obtained results are checked with the test results from the literature.

3. Analysis

I. Linear and nonlinear regression analysis

The following different models are tested to find the best equations representing cracking and ultimate punching shear of slabs in terms of the variables (flexural reinforcement index ρ, steel fiber properties, and slab dimension), using the experimental database found in literature and shown in Table 1.

Linear formula: $y = a + bx$ (1)

Power formula: $y = ax^{b}$ (2)

Exponential formula: $y = ae^{bx}$ (3)

Table 1: Experimental data for punching shear of concrete slabs with steel fiber

No.	Reference	V_f %	F	d (mm)	$b_o d$ (mm^2)	f_c' (Mpa)	f_{sp} (Mpa)	ρ %	ρf_y	V_{cr} (KN)	V_u (KN)
1	Ref. [21]	0.4	0.32	105	107100	22.32	2.23	0.66	3.2472	30	330
2		0.6	0.48	105	107100	23.36	2.42	0.66	3.2472	40	345
3		0.8	0.64	105	107100	25.28	2.57	0.66	3.2472	45	397
4		0.4	0.32	105	107100	22.32	2.23	0.66	3.2472	35	328
5		0.6	0.48	105	107100	23.36	2.42	0.66	3.2472	40	337
6		0.8	0.64	105	107100	25.28	2.57	0.66	3.2472	45	347
7		0.4	0.32	105	107100	22.32	2.23	0.66	3.2472	46	307
8		0.6	0.48	105	107100	23.36	2.42	0.66	3.2472	50	310
9		0.8	0.64	105	107100	25.28	2.57	0.66	3.2472	55	326
10	Ref. [18]	1	0.55	127	141732	25.4	3.15	0.83	3.7765		386
11		1	0.55	127	141732	25.4	3.15	0.56	2.548		389
12		1.5	0.84	127	141732	59.3	4.81	0.83	3.9093		530
13		1.5	0.84	127	141732	57.9	4.76	0.56	2.6376		444
14		1.5	0.82	127	141732	31	3.48	0.83	3.7267		522
15		1.5	0.82	127	141732	31	3.48	0.56	2.5144		472
16		1.5	1.19	127	141732	46.1	4.24	0.83	3.7267		530
17		1.5	1.19	127	141732	59.1	4.80	0.56	2.5144		503
18	Ref. [17]	0.75	0.38	80	57600	44	4.15	0.63	2.25		313
19	Ref. [14]	0.5	0.33	40	12800	82.16	5.67	0.94	3.393		65
20		1	0.65	40	12800	90.56	5.95	0.94	3.393		85
21	Ref. [13]	0.31	0.31	21.8	10621	46.2	4.25	1.63	1.14	6.7	21.4
22		0.31	0.31	21.8	10621	45.8	4.23	1.63	1.14	5.5	22.6
23		0.31	0.31	21.8	10621	47.2	4.29	1.63	1.14	5.3	18.9
24		0.5	0.5	21.8	10621	40.3	3.97	1.63	1.14	6.6	20.9
25		1	1	21.8	10621	40.7	3.99	1.63	1.14	5.1	23.7
26		1.5	1.5	21.8	10621	39.7	3.94	1.63	1.14	4.5	24.6
27		2	2	21.8	10621	47.8	4.32	1.63	1.14	9.1	27.4
28		0.31	0.31	13.7	6131	46.9	4.28	2.45	1.712	3.1	9.4
29		0.31	0.31	35.5	19241	46.1	4.24	0.94	0.66	15.5	54.9
30		0.31	0.31	43.6	25044	48.4	4.35	0.77	0.538	23.9	70.5
31		0.31	0.31	21.8	10621	37.6	3.83	1.63	1.14	5.5	19
32		0.31	0.31	21.8	10621	60.6	4.87	1.63	1.14	7	20
33		0.31	0.31	21.8	10621	41.4	4.02	1.63	1.14	6.2	26.1
34		0.31	0.31	21.8	10621	39.8	3.94	1.63	1.14	5.3	18.7
35	Ref. [12]	1	0.55	80	51200	24.4	2.59	1.57	5.652		139.6
36		2	1.09	80	51200	28.1	2.98	1.57	5.652		163.6
37		1	0.55	80	51200	59.7	5.45	1.57	5.652		215.1
38		2	1.09	80	51200	52.4	6.59	1.57	5.652		236.2
39		0.75	0.37	80	51200	36.6	3.97	1.57	5.652		182.9
40		1.5	0.72	80	51200	46.1	5.17	1.57	5.652		210.9
41	Ref. [9]	1	1	58	62691	100.8	6.27	0.33	1.386		459.3
42		1	1	58	65828	100.8	6.27	0.66	2.772		620.4
43		1	1	40	30410	100.8	6.27	0.66	2.772		296.6
44		1	1	40	28390	100.8	6.27	0.33	1.386		236.2
45		2	2	58	61712	118	6.79	0.66	2.772		790.8
46		2	2	40	28678	118	6.79	0.66	2.772		387
47		2	2	40	26432	118	6.79	0.33	1.386		241.3
48		2	2	58	58650	118	6.79	0.33	1.386		558
49		2	2	58	64023	105.3	6.41	0.66	2.772		730.1
50		2	2	58	60097	105.3	6.41	0.33	1.386		486.4
51		2	2	40	29437	105.3	6.41	0.66	2.772		377.8
52		2	2	40	27430	105.3	6.41	0.33	1.386		228.8
53	Ref. [24]	0.6	0.72	105	107100	34.3	3.66	0.5	2.1		244
54		0.9	1.08	105	107100	34.3	3.66	0.5	2.1		263
55		1.2	1.44	105	107100	34.3	3.66	0.5	2.1		281
56		0.9	1.08	105	107100	34.3	3.66	0.5	2.1		267
57		0.9	1.08	105	107100	34.3	3.66	0.5	2.1		239
58		0.9	0.37	105	107100	34.3	3.66	0.66	2.772		237
59		0.9	0.9	105	107100	34.3	3.66	0.66	2.772		249
60		0.9	1.08	105	107100	34.3	3.66	0.66	2.772		262
61		0.9	1.08	105	107100	34.3	3.66	0.66	2.772		256
62		0.9	1.08	105	107100	34.3	3.66	0.5	2.1		213
63		0.9	1.08	105	107100	34.3	3.66	0.42	1.764		203
64		0.9	1.08	105	107100	34.3	3.66	0.33	1.386		179
65	Ref. [25]	0.5	0.6	100	100000	29.9	3.42	0.56	2.352		225
66		1	1.2	100	100000	31.4	3.50	0.56	2.352		247
67		1	1.2	100	100000	32.9	3.58	0.56	2.352		224
68		1	1.2	100	100000	33.5	3.62	0.37	1.554		198
69		1	1.2	100	100000	31.4	3.50	0.37	1.554		175
70		1	1.2	100	100000	32.3	3.55	0.37	1.554		192
71		1	1.2	100	100000	32.6	3.57	0.37	1.554		211
72		1	1.2	100	80000	31.3	3.50	0.56	2.352		217
73		1	1.2	100	120000	30.1	3.43	0.56	2.352		260
74		1	0.6	100	100000	31.8	3.52	0.56	2.352		218
75		1	1	100	100000	29.5	3.39	0.56	2.352		236
76		1	0.84	100	100000	30.8	3.47	0.56	2.352		240
77		1	0.9	100	100000	27.5	3.28	0.56	2.352		238
78		1	0.7	100	100000	24.6	3.10	0.56	2.352		228
79		1	0.7	100	100000	41.2	4.01	0.56	2.352		268
80		1	0.7	100	100000	12.5	2.21	0.56	2.352		166
81	Ref. [26]	0.45	0.45	39	21684	30	3.42	1.12	4.704		68
82		0.8	0.8	39	21684	31.4	3.50	1.12	4.704		78
83		1	0.6	39	21684	24.6	3.10	1.12	4.704		69
84		2	1.2	39	21684	20	2.80	1.12	4.704		62
85		0.45	0.45	55	34100	31.4	3.50	1.12	4.704		115
86		0.8	0.8	55	34100	31.8	3.52	1.12	4.704		117
87		1	0.6	55	34100	29.1	3.37	1.12	4.704		118
88		2	1.2	55	34100	29.2	3.38	1.12	4.704		146
89	Ref. [27]	1.3	0.38	138	186576	35.8	3.74	0.43	1.806		324
90		2.7	0.78	138	186576	35	3.70	0.43	1.806		345

Table 1: Continued

No.	Reference	V_f %	F	d (mm)	$b_o d$ (mm^2)	f_c' (Mpa)	f_{sp} (Mpa)	ρ %	ρf_y	V_{cr} (KN)	V_u (KN)
	Ref. [21]	0.4	0.32	105	107100	22.32	2.23	0.66	3.2472	30	330
91		1.4	0.41	111	138084	38.4	3.87	0.54	2.268		308
92		2.8	0.81	111	138084	38.5	3.88	0.54	2.268		330
93	Ref. [28]	0.5	0.36	109	145624	41.5	4.03	1.12	4.704		422
94		0.5	0.36	109	145624	41.5	4.03	2.18	9.156		438
95	Ref. [29]	0.25	0.25	90	54000	103	6.34	0.87	3.654		318
96		0.51	0.51	90	54000	108	6.50	0.87	3.654		343
97		0.76	0.76	90	54000	106	6.43	0.87	3.654		337
98		1.02	1.02	90	54000	107	6.47	0.87	3.654		369
99		0.51	0.51	92	55936	108	6.50	0.55	2.31		286
100		1.02	1.02	92	55936	107	6.47	0.55	2.31		327
101		0.51	0.51	88	52096	108	6.50	1.29	5.418		361
102		1.02	1.02	88	52096	107	6.47	1.29	5.418		402
103	Ref. [30]	0.5	0.33	44	25344	35.4	4.34	1.5	10.05	10	89.5
104		0.5	0.33	44	25344	49.1	4.90	1.5	10.05	14.75	102.5
105		0.5	0.33	44	25344	55.1	5.20	1.5	10.05	16	129.5
106		0.5	0.33	44	25344	65.1	6.12	1.5	10.05	20	141
107		0.25	0.17	44	25344	48.8	4.60	1.5	10.05	14	98
108		0.75	0.5	44	25344	52.4	6.30	1.5	10.05	17	125.5
109		1	0.67	44	25344	53.3	6.70	1.5	10.05	19.5	138
110		0.5	0.13	44	25344	49.2	4.08	1.5	10.05	13	98
111		0.5	0.17	44	25344	49.2	4.10	1.5	10.05	17	100
112		0.5	0.21	44	25344	52.8	5.10	1.5	10.05	19	117.5
113		0.5	0.25	44	25344	49.5	4.50	1.5	10.05	18	110
114		0.5	0.33	44	20944	51.1	4.84	1.5	10.05	14.5	88.5
115		0.5	0.33	44	34144	50.8	4.72	1.5	10.05	15	135
116	Ref. [31]	0.25	0.25	45	26100	52.1	4.51	1.84	7.728		93.4
117		0.5	0.5	45	26100	44.7	4.18	1.84	7.728		102
118		0.75	0.75	45	26100	46	4.24	1.84	7.728		107.5
119		1	1	45	26100	53	4.55	1.84	7.728		113.6
120		1.25	1.25	45	26100	53	4.55	1.84	7.728		122.2
121		1	1	45	26100	47	4.28	1.6	6.72		92.6
122		1	1	45	26100	45.3	4.21	2.08	8.736		111.1
123		1	1	45	26100	43.5	4.12	2.3	9.66		111.3
124		1	1	45	26100	47.6	4.31	2.53	10.626		111.3
125		1	1	45	26100	29.8	3.41	1.84	7.728		82.1
126		1	1	45	26100	32.4	3.56	1.84	7.728		84.9
127	Ref. [20]	0.67	0.32	70	47600	24.6	3.10	0.85	3.57		137.5
128		0.67	0.32	110	92400	24.6	3.10	0.54	2.268		210.2
129		0.67	0.32	150	150000	24.6	3.10	0.4	1.68		297.6
130		0.72	0.35	65	42900	42.4	4.07	0.91	3.822		140.8
131		0.72	0.35	105	86100	42.4	4.07	0.57	2.394		213.2
132		0.72	0.35	145	142100	42.4	4.07	0.41	1.722		290.7
133		0.91	0.44	65	42900	21.6	2.90	0.91	3.822		120.8
134		0.91	0.44	110	92400	21.6	2.90	0.57	2.394		183.1
135		0.91	0.44	145	142100	21.6	2.90	0.41	1.722		231.2
136		0.63	0.3	70	47600	27.6	3.28	0.85	3.57		152.3
137		0.94	0.45	70	47600	31.1	3.49	0.85	3.57		147.9
138		1.03	0.5	70	47600	30.4	3.45	0.85	3.57		158.9
139	Ref. [32]	1	1	100	120000	19.6	2.77	0.98	4.116		290
140		1.5	1.5	100	120000	20.2	2.81	0.98	4.116		315
141		1	1	100	120000	15	2.42	0.98	4.116		285
142		1.5	1.5	100	120000	14.9	2.41	0.98	4.116		310
143	Ref. [33]	1	1	75	58500	51.2	4.47	0.95	3.99		226.5
144		1	1	75	58500	51.2	4.47	0.86	3.612		204
145		1	1	75	58500	51.2	4.47	0.76	3.192		212.8
146		1	1	75	58500	51.2	4.47	0.67	2.814		178.5
147		1.5	1.5	75	58500	60.8	4.87	0.95	3.99		249.1
148		1.5	1.5	75	58500	60.8	4.87	0.86	3.612		203
149		1.5	1.5	75	58500	60.8	4.87	0.76	3.192		222.6
150		1.5	1.5	75	58500	60.8	4.87	0.67	2.814		204
151		2	2	75	58500	50.3	4.43	0.95	3.99		263.8
152		2	2	75	58500	50.3	4.43	0.86	3.612		223.6
153		2	2	75	58500	50.3	4.43	0.76	3.192		203
154		2	2	75	58500	50.3	4.43	0.67	2.814		205
155	Ref. [34]	0.61	0.61	60	29640	34.5	3.67	1.56	6.552		94.5
156		0.61	0.61	60	29640	37.3	3.82	1.56	6.552		112.5
157		0.61	0.61	60	29640	29.7	3.41	1.56	6.552		72
158		0.92	0.92	60	29640	37.7	3.84	1.56	6.552		108
159		1.22	1.22	60	29640	46.8	4.28	1.56	6.552		135
160		1.22	1.22	60	29640	36.6	3.78	1.56	6.552		117
161		1.84	1.84	60	29640	22.4	2.96	1.56	6.552		99
162		1.93	1.93	60	29640	22.1	2.94	1.56	6.552		103.5
163	Ref. [35]	0.5	0.5	58	27376	48.5	4.35	1.37	5.754		94.1
164		1	1	58	27376	49.2	4.38	1.37	5.754		111.5
165		1.5	1.5	58	27376	49	4.38	1.37	5.754		114.6
166	Ref. [36]	1	1	60	38400	28.03	3.31	0.65	2.7342	35	85
167		1	1	60	38400	28.03	3.31	0.76	3.1962	45	91.4
168		1	1	60	38400	28.03	3.31	0.91	3.8304	55	117.5
169		1	1	60	38400	28.03	3.31	1.14	4.788	60	142.5
170		1	1	60	38400	28.03	3.31	1.52	6.384	70	148.5
171		0.25	0.25	60	38400	28.01	3.31	0.91	3.8304	45	95
172		0.5	0.5	60	38400	28.21	3.32	0.91	3.8304	50	102.5
173		0.75	0.75	60	38400	28.55	3.34	0.91	3.8304	55	113
174		1.25	1.25	60	38400	27.19	3.26	0.91	3.8304	57.5	110
175		1.5	1.5	60	38400	25.33	3.15	0.91	3.8304	55	99.35
176		1	1	60	38400	10.53	2.03	0.91	3.8304	25	72
177		1	1	60	38400	15.81	2.49	0.91	3.8304	35	82
178		1	1	60	38400	16.37	2.53	0.91	3.8304	40	96.5
179		1	1	60	38400	19.04	2.73	0.91	3.8304	47.5	108.4
180		1	1	60	38400	34.18	3.65	0.91	3.8304	55	130

Reciprocal formula 1: $$y = \frac{1}{a + bx} \tag{4}$$

Reciprocal formula 2: $$y = \frac{x}{a + bx} \tag{5}$$

Where y takes the value of cracking punching shear (V_{cr}) or Ultimate punching shear (V_u) and x takes the value of flexural reinforcement index (ρ) or (ρf_y) $a \& b$ are coefficient are determined using the data shown in Table 1 and special programs of regression analysis. (Linear and nonlinear regression analysis). The results of the analysis are shown below:

Unit of V_u is (kN)

$$V_u = 324.5 - 115\rho \tag{6}$$

$$V_u = 151\rho^{-0.77} \tag{7}$$

$$V_u = 358.5e^{-0.77\rho} \tag{8}$$

$$V_u = \frac{1}{-0.0012 + 0.01\rho} \tag{9}$$

$$V_u = \frac{\rho}{-0.006 + 0.0164\rho} \tag{10}$$

Unit of V_{cr} is (kN)

$$V_u = 227 - 3.9\rho f_y \tag{11}$$

$$V_u = 162(\rho f_y)^{0.024} \tag{12}$$

$$V_u = 170\rho^{-0.005\rho f_y} \tag{13}$$

$$V_u = \frac{1}{0.01 - 0.0003\rho f_y} \tag{14}$$

$$V_u = \frac{\rho f_y}{0.018 + 0.0025\rho f_y} \tag{15}$$

$$V_{cr} = 42 - 6.6\rho \tag{16}$$

$$V_{cr} = 28\rho^{-0.98} \tag{17}$$

$$V_{cr} = 66e^{-0.8\rho} \tag{18}$$

$$V_{cr} = \frac{1}{-0.03 + 0.07\rho f_y} \tag{19}$$

$$V_{cr} = \frac{\rho}{-0.086 + 0.14\rho} \tag{20}$$

$$V_{cr} = 23.3 + 1.7\rho f_y \tag{21}$$

$$V_{cr} = 13\rho f_y^{0.46} \tag{22}$$

$$V_{cr} = 19e^{0.044\rho f_y} \tag{23}$$

$$V_{cr} = \frac{1}{0.078 - 0.003\rho f_y} \tag{24}$$

$$V_{cr} = \frac{\rho f_y}{0.099 + 0.005\rho f_y} \tag{25}$$

The results show that there is no good correlation between cracking & ultimate punching shear with the flexural reinforcement (ρ & ρf_y), Therefore another equations are proposed taking concrete compressive strength ($f_c^{'}$), critical section perimeter at $(d/2)$ from the column face (b_o) and the effective depth of concrete section into accounts on this basis the following equations are determined and shown below:

$$V_u = [0.567 - 0.03\rho] b_o d \sqrt{f_c^{'}} / 1000 \tag{26}$$

$$V_u = [5.157 x 10^{-4} \rho^{0.158}] b_o d \sqrt{f_c^{'}} \tag{27}$$

$$V_u = 5.39 x 10^{-4} e^{-0.063\rho}] b_o d \sqrt{f_c^{'}} \tag{28}$$

$$V_u = [\frac{1}{1.873 + 0.242\rho}] b_o d \sqrt{f_c^{'}} / 1000 \tag{29}$$

$$V_u = [\frac{\rho}{-0.0002 + 2.115\rho}] b_o d \sqrt{f_c^{'}} / 1000 \tag{30}$$

$$V_u = [0.558 - 0.005\rho f_y] b_o d \sqrt{f_c^{'}} / 1000 \tag{31}$$

$$V_u = [4.735 x 10^{-4} (\rho f_y)^{0.053}] b_o d \sqrt{f_c^{'}} \tag{32}$$

$$V_u = [5.455 x 10^{-4} e^{-0.0167\rho f_y}] b_o d \sqrt{f_c^{'}} \tag{33}$$

$$V_u = [\frac{1}{1.748 + 0.082\rho f_y}] b_o d \sqrt{f_c^{'}} / 1000 \tag{34}$$

$$V_u = [\frac{\rho f_y}{0.743 + 1.856\rho f_y}] b_o d \sqrt{f_c^{'} / 1000} \tag{35}$$

$$V_{cr} = [0.215 + 0.063\rho] b_o d \sqrt{f_c^{'}} / 1000 \tag{36}$$

$$V_{cr} = [1.213 x 10^{-4} \rho^{-0.457}] b_o d \sqrt{f_c^{'}} \tag{37}$$

$$V_{cr} = [2.07 x 10^{-4} e^{-0.485\rho}] b_o d \sqrt{f_c^{'}} \tag{38}$$

$$V_{cr} = [\frac{1}{4.5 + 4.89\rho}] b_o d \sqrt{f_c^{'}} / 1000 \tag{39}$$

$$V_{cr} = [\frac{\rho}{-0.34 + 1.368\rho}] b_o d \sqrt{f_c^{'}} / 10000 \tag{40}$$

$$V_{cr} = [0.16 - 0.0037\rho f_y] b_o d \sqrt{f_c^{'}} / 1000 \tag{41}$$

$$V_{cr} = [1.348 x 10^{-4} (\rho f_y)^{-0.114}] b_o d \sqrt{f_c^{'}} \tag{42}$$

$$V_{cr} = [1.443 x 10^{-4} e^{-0.0304\rho f_y}] b_o d \sqrt{f_c^{'}} \tag{43}$$

$$V_{cr} = [\frac{1}{7.097 + 0.548\rho f_y}] b_o d \sqrt{f_c^{'}} / 1000 \tag{44}$$

$$V_{cr} = [\frac{\rho f_y}{-0.139 + 1.095\rho f_y}] b_o d \sqrt{f_c^{'}} / 1000 \tag{45}$$

Equations 26, 27, 28, 31, 32 & 33 can be used to predict ultimate punching shear with good agreement with the experimental data, and equations 36 & 41 can be used for cracking punching shear.

II.Multi-linear regression analysis

Based on the equilibrium of the forces acts on the failure surface shown in Figure 1 at distance ($d / 2$) from the face of the column, the general equation of (V_u) can be written as:

$$V_u = V_c + V_f \tag{46}$$

$$V_u = K_1 \sqrt{f_c^{'}} b_o d + K_2 \sigma f_u b_o d \tag{47}$$

Where the 1st term represents shear carried by the concrete and the 2nd term is the shear carried by the steel fibers.

σf_u = Ultimate tensile strength of the fibrous concrete (Mpa), calculated as the following [37]

$$\sigma f_u = 0.82\tau F \qquad (48)$$

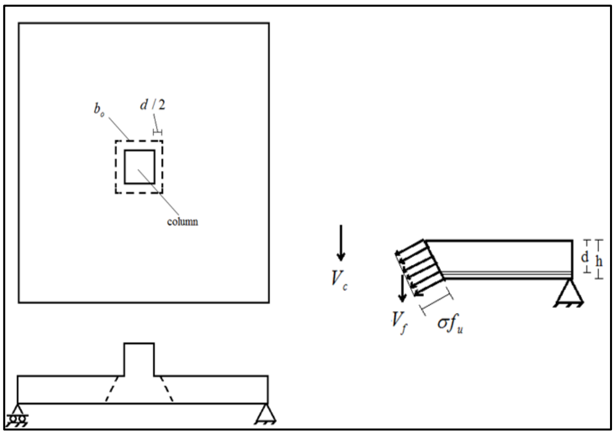

Figure 1: Suggested failure surface

Where τ is the interfacial bond strength between steel fiber and concrete matrix (Mpa), and can be taken equal to (4.15Mpa) [11].

Using multi-linear regression analysis and experimental database found in the literature Table 1, the value of the coefficients (K_o & K_1) are determined.

$$\text{F = Fiber factor [39\&40]} = Q_f \cdot \frac{L}{d} . d_f \qquad (49)$$

Where Q_f is the steel fiber content by volume (%).

L/d is the aspect ratio of steel fiber (L = length & d = diameter of steel fiber (mm).

d_f is the bond factor, depends on the type of steel fiber, its value vary between (0.9 & 1.2) [38].

In addition, the effect of the flexural reinforcement index is included as the 3rd term in eq.47; the final equation becomes the following:

$$V_u = (K_1\sqrt{f_c^{'}} + K_2\sigma f_u + K_3\rho_{fy})b_{od} \qquad (50)$$

The following equations are determined for the ultimate punching shear (V_u).

$$V_u = [0.476 + 0.077Q_f]\, b_o d\sqrt{f_c^{'}}\,/1000 \qquad (51)$$

$$V_u = [0.474 + 0.027\sigma f_u]\, b_o d\sqrt{f_c^{'}}\,/1000 \qquad (52)$$

$$V_u = [0.504\sqrt{f_c^{'}} + 0.258Q_f]\, b_o d\,/1000 \qquad (53)$$

$$V_u = [0.5014\sqrt{f_c^{'}} + 0.0964\sigma f_u]\, b_o d\,/1000 \qquad (54)$$

$$V_u = [0.1116 f_{sp} + 0.0276\rho f_y]\, b_o d\sqrt{f_c^{'}}\,/1000 \qquad (55)$$

$$V_u = [0.125 f_{sp} + 0.085Q_f]\, b_o d\sqrt{f_c^{'}}\,/1000 \qquad (56)$$

Where $f_{sp} =$ split tensile strength of the concrete (Mpa).

$$V_u = [0.099 f_{sp} + 0.167\rho]\, b_o d\sqrt{f_c^{'}}\,/1000 \qquad (57)$$

$$V_u = [0.127 f_{sp} + 0.026\sigma f_u]\, b_o d\sqrt{f_c^{'}}\,/1000 \qquad (58)$$

Taking three terms, concrete, steel fiber & flexural reinforcement contributions, the following equations are proposed.

$$V_u = [0.925 f_{sp} + 0.285 \rho f_y + 0.9 Qf\ ]\, b_o d \sqrt{f_c^{'}} 10000 \quad (59)$$

$$V_u = [0.935 f_{sp} + 0.29 \rho f_y + 0.286 \sigma f_u] b_o d \sqrt{f_c^{'}} 10000 \quad (60)$$

$$V_u = [0.494 - 0.0036 \rho f_y + 0.075 Qf\ ] b_o d \sqrt{f_c^{'}} 10000 \quad (61)$$

$$V_u = [0.49 - 0.003 \rho f_y + 0.0265 \sigma f_u] b_o d \sqrt{f_c^{'}} 10000 \quad (62)$$

Ultimate punching shear can be determined from the previous equations (51-62) with acceptable agreement with experimental data.

The same procedures are applied on cracking punching shear and the following equations are determined.

$$V_{cr} = [0.2934 f_{sp} + 0.019 \rho f_y + 0.758 Q_f\ ] b_o d \sqrt{f_c^{'}} / 10000 \quad (63)$$

$$V_{cr} = [0.255 f_{sp} + 0.036 \rho f_y + 0.279 \sigma f_u] b_o d \sqrt{f_c^{'}} / 10000 \quad (64)$$

$$V_{cr} = [0.009 f_{sp} + 0.106 \rho] b_o d \sqrt{f_c^{'}} / 1000 \quad (65)$$

$$V_{cr} = [0.314 f_{sp} + 0.263 \sigma f_u] b_o d \sqrt{f_c^{'}} / 10000 \quad (66)$$

$$V_{cr} = [0.139 f_{sp} - 0.0036 \rho f_y + 0.039 Q_f\ ] b_o d \sqrt{f_c^{'}} / 1000 \quad (67)$$

$$V_{cr} = [0.116 + 0.0416 Q_f\ ] b_o d \sqrt{f_c^{'}} / 1000 \quad (68)$$

$$V_{cr} = [0.11 + 0.0176 \sigma f_u] b_o d \sqrt{f_c^{'}} / 1000 \quad (69)$$

$$V_{cr} = [0.106 \sqrt{f_c^{'}} + 0.353 Q_f\ ] b_o d\ / 1000 \quad (70)$$

$$V_{cr} = [0.1043 \sqrt{f_c^{'}} + 0.122 \sigma f_u] b_o d\ / 1000 \quad (71)$$

$$V_{cr} = [0.444 f_{sp} - 0.0082 \rho f_y] b_o d \sqrt{f_c^{'}} / 10000 \quad (72)$$

$$V_{cr} = [0.324 f_{sp} + 0.736 Q_f\ ] b_o d \sqrt{f_c^{'}} / 10000 \quad (73)$$

The accuracy of determining (V_{cr}) is not as the ultimate punching shear (V_u), because the number of available data is less, the best-fit equations are (65, 68, and 72), where the results of (R_{avg}) can be accepted.

III. Multi Non-linear regression analysis

Nonlinear equations are proposed to predict cracking and ultimate punching shear, which take the following form:

$$V_u = [\alpha_o f_c^{'\alpha_1} (\rho f_y)^{\alpha_2} d^{\alpha_3} F^{\alpha_4}]\, b_o d \quad (74)$$

The above equation transformed to the multi-linear equation by taking log of both sides.

$$\log(\frac{V_u}{b_o d}) = \log \alpha_o + \alpha_1 \log f_c^{'} + \alpha_2 \log(\rho f_y) + \alpha_3 \log d + \alpha_4 \log F$$

Taking $y_1 = \log(\frac{V_u}{b_o d}),\ \ x_1 = \log f_c^{'},\ \ x_2 = \log \rho f_{y,}\ \ x_3 = \log d \ \ \& \ x_4 = \log F$

The final term becomes:

$$y_1 = K_o + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 \quad (75)$$

Using multi-linear regression analysis and applying on the available experimental data, Table 1, value of the coefficients $\alpha_o = e^{ko},\ \alpha_1, \alpha_2, \alpha_3 \,\&\, \alpha_4$ are determined.

$$V_u = [8.553\, x\, 10^{-4} f_c^{'0.4424} (\rho f_y)^{0.126} d^{-0.0934} F^{0.1906}]\, b_o d \quad (76)$$

Neglecting the term of (d) ; the above becomes as the following:

$$V_u = 5.8 x 10^{-4} f_c^{'0.46} (\rho f_y)^{0.12} F^{0.18}]\, b_o d \quad (77)$$

Taking the square root of (f_c'); the equation becomes:

$$V_u = [5x\,10^{-4}(\rho f_y)^{0.12} F^{0.18}]\, b_o d \sqrt{f_c'} \qquad (78)$$

Taking (ρ) instead of (ρf_y);

Also; $$V_u = [8.334x\,10^{-4} f_c'^{0.471} \rho^{0.008} d^{-0.0764} F^{0.1677}]\, b_o d \qquad (79)$$

Neglecting the term of (d); the equation becomes:

$$V_u = [6.25\,x\,10^{-4} f_c'^{0.48} \rho^{0.05} F^{0.17}]\, b_o d \qquad (80)$$

Taking square root of (f_c')

$$V_u = [5.8\;x\;10^{-4} \rho^{0.05} F^{0.17}]\, b_o d \sqrt{f_c'} \qquad (81)$$

Some procedures are applied for cracking punching shear, and the following equations are obtained:

$$V_{cr} = [1.95\;x\;10^{-4}(\rho f_y)^{-0.05} F^{0.5}]\, b_o d \sqrt{f_c'} \qquad (82)$$

$$V_{cr} = [1.9\;x\;10^{-4}(\rho)^{-0.35} F^{0.5}]\, b_o d \sqrt{f_c'} \qquad (83)$$

Values of (V_u) calculated from equations (77, 78, 80, & 81) are shown in Table 2, and the plot of ($V_{u\,exp.}$ verse $V_{u\,cal.}$) for these equations are shown in Figures 2, 3, 4 & 5.

The results are compared with ACI equation [41]:

$$V_u = \frac{1}{3}\sqrt{f_c'}\, b_o d \qquad (84)$$

The calculated results from this equation give underestimated results, where results from the proposed equations give more economical and nearest to the experimental data where (R_{avg}) reached to unity value.

Values of (V_{cr}) calculated from equations (82 & 83) are shown in Table 3; Also, the values calculated using ACI-Code equation (84) and the additional ACI-Code equation, which is used for one way shear action as shown below:

$$V_u = \frac{1}{6}\sqrt{f_c'}\, b_o d \qquad (85)$$

The results obtained from equations (84 & 85) are overestimated, for determining cracking punching shear, and give poor correlation with the experimental data, while the proposed equations (82 & 83) give good agreement and acceptable correlation with the experimental data, the plot of ($V_{cr\,exp.}$ verse $V_{cr\,cal.}$) for equation (82 & 83) are shown in Figures 6 & 7.

Table 2: Predicted results for ultimate punching shear of concrete slabs with steel fiber

$$V_u = 5.8x\,10^{-4}\,(f_c')^{0.46}\,(\rho f_y)^{0.12}\,F^{0.18}\,b_o d \qquad (1)$$

$$V_u = 5x\,10^{-4}\,(f_c')^{0.5}\,(\rho f_y)^{0.12}\,F^{0.18}\,b_o d \qquad (2)$$

$$V_u = 6.25x\,10^{-4}\,(f_c')^{0.48}\,(\rho)^{0.05}\,F^{0.17}\,b_o d \qquad (3)$$

$$V_u = 5.8x\,10^{-4}\,(f_c')^{0.5}\,(\rho)^{0.05}\,F^{0.17}\,b_o d \qquad (4)$$

$$V_u = 0.333x\,10^{-3}\,(f_c')^{0.5}\;b_o d \;\text{(ACI – Code)} \qquad (5)$$

			Eq.(1) 77		Eq.(2) 78		Eq.(3) 80		Eq.(4) 81		Eq.(5) ACI-Code	
No.	Reference	$V_{u\,\exp}$ (KN)	$V_{u\,cal}$ (KN)	$R = \frac{V_{u\,\exp}}{V_{u\,cal}}$	$V_{u\,cal}$ (KN)	$R = \frac{V_{u\,\exp}}{V_{u\,cal}}$	$V_{u\,cal}$ (KN)	$R = \frac{V_{u\,\exp}}{V_{u\,cal}}$	$V_{u\,cal}$ (KN)	$R = \frac{V_{u\,\exp}}{V_{u\,cal}}$	$V_{u\,cal}$ (KN)	$R = \frac{V_{u\,\exp}}{V_{u\,cal}}$
1	Ref. [21]	330	243.178	1.357	237.364	1.390	239.826	1.376	236.819	1.393	168.661	1.957
2		345	267.128	1.292	261.216	1.321	262.618	1.314	259.563	1.329	172.546	1.999
3		397	291.735	1.361	286.181	1.387	286.438	1.386	283.553	1.400	179.497	2.212
4		328	243.178	1.349	237.364	1.382	239.826	1.368	236.819	1.385	168.661	1.945
5		337	267.128	1.262	261.216	1.290	262.618	1.283	259.563	1.298	172.546	1.953
6		347	291.735	1.189	286.181	1.213	286.438	1.211	283.553	1.224	179.497	1.933
7		307	243.178	1.262	237.364	1.293	239.826	1.280	236.819	1.296	168.661	1.820
8		310	267.128	1.160	261.216	1.187	262.618	1.180	259.563	1.194	172.546	1.797
9		326	291.735	1.117	286.181	1.139	286.438	1.138	283.553	1.150	179.497	1.816
10	Ref. [18]	386	383.385	1.007	376.158	1.026	374.527	1.031	370.790	1.041	238.102	1.621
11		389	365.703	1.064	358.810	1.084	367.231	1.059	363.566	1.070	238.102	1.634
12		530	613.654	0.864	622.856	0.851	604.638	0.877	608.842	0.871	363.809	1.457
13		444	578.953	0.767	587.074	0.756	586.098	0.758	589.891	0.753	359.489	1.235
14		522	450.783	1.158	445.825	1.171	441.066	1.183	438.409	1.191	263.043	1.984
15		472	429.993	1.098	425.263	1.110	432.473	1.091	429.867	1.098	263.043	1.794
16		530	578.131	0.917	580.920	0.912	568.081	0.933	569.157	0.931	320.772	1.652
17		503	618.226	0.814	627.412	0.802	627.554	0.802	631.875	0.796	363.195	1.385

18	Ref. [17]	313	175.965	1.779	176.484	1.774	183.037	1.710	183.213	1.708	127.358	2.458
19	Ref. [14]	65	53.357	1.218	54.868	1.185	54.684	1.189	55.425	1.173	38.674	1.681
20		85	63.215	1.345	65.259	1.302	64.466	1.319	65.466	1.298	40.603	2.093
21	Ref. [13]	21.4	29.553	0.724	29.698	0.721	35.093	0.610	35.161	0.609	24.064	0.889
22		22.6	29.435	0.768	29.569	0.764	34.946	0.647	35.008	0.646	23.959	0.943
23		18.9	29.846	0.633	30.018	0.630	35.455	0.533	35.539	0.532	24.323	0.777
24		20.9	30.247	0.691	30.230	0.691	35.647	0.586	35.619	0.587	22.475	0.930
25		23.7	34.422	0.689	34.416	0.689	40.296	0.588	40.272	0.589	22.586	1.049
26		24.6	36.607	0.672	36.564	0.673	42.659	0.577	42.612	0.577	22.307	1.103
27		27.4	41.990	0.653	42.254	0.648	48.973	0.559	49.101	0.558	24.477	1.119
28		9.4	18.037	0.521	18.136	0.518	20.824	0.451	20.871	0.450	13.996	0.672
29		54.9	50.090	1.096	50.331	1.091	61.797	0.888	61.914	0.887	43.547	1.261
30		70.5	65.058	1.084	65.499	1.076	81.496	0.865	81.730	0.863	58.077	1.214
31		19	26.881	0.707	26.792	0.709	31.789	0.598	31.720	0.599	21.709	0.875
32		20	33.481	0.597	34.013	0.588	39.974	0.500	40.269	0.497	27.560	0.726
33		26.1	28.099	0.929	28.113	0.928	33.293	0.784	33.284	0.784	22.780	1.146
34		18.7	27.594	0.678	27.565	0.678	32.669	0.572	32.634	0.573	22.335	0.837
35	Ref. [12]	139.6	142.467	0.980	139.557	1.000	136.798	1.020	135.324	1.031	84.303	1.655
36		163.6	172.228	0.950	169.666	0.964	164.697	0.993	163.384	1.001	90.469	1.809
37		215.1	215.012	1.001	218.294	0.986	210.184	1.024	211.674	1.016	131.867	1.631
38		236.2	229.399	1.030	231.690	1.019	222.119	1.063	223.111	1.059	123.542	1.912
39		182.9	160.118	1.142	159.412	1.147	155.600	1.175	155.177	1.178	103.250	1.771
40		210.9	200.717	1.051	201.685	1.046	194.656	1.083	195.025	1.081	115.878	1.820
41	Ref. [9]	459.3	315.672	1.455	327.278	1.403	339.369	1.353	345.374	1.330	209.804	2.189
42		620.4	360.218	1.722	373.462	1.661	368.917	1.682	375.445	1.652	220.303	2.816
43		296.6	166.407	1.782	172.525	1.719	170.426	1.740	173.441	1.710	101.771	2.914
44		236.2	142.954	1.652	148.210	1.594	153.685	1.537	156.405	1.510	95.011	2.486
45		790.8	411.324	1.923	429.142	1.843	419.667	1.884	428.441	1.846	223.455	3.539
46		387	191.145	2.025	199.425	1.941	195.022	1.985	199.099	1.944	103.841	3.727
47		241.3	162.114	1.488	169.137	1.426	173.626	1.390	177.256	1.361	95.708	2.521
48		558	359.715	1.551	375.298	1.487	385.258	1.448	393.313	1.419	212.367	2.627
49		730.1	404.950	1.803	420.572	1.736	412.225	1.771	419.885	1.739	218.992	3.334
50		486.4	349.780	1.391	363.274	1.339	373.766	1.301	380.711	1.278	205.563	2.366
51		377.8	186.191	2.029	193.374	1.954	189.536	1.993	193.058	1.957	100.690	3.752
52		228.8	159.650	1.433	165.809	1.380	170.597	1.341	173.768	1.317	93.825	2.439
53	Ref. [24]	244	325.416	0.750	323.141	0.755	333.661	0.731	332.322	0.734	209.081	1.167
54		263	350.054	0.751	347.607	0.757	357.471	0.736	356.037	0.739	209.081	1.258
55		281	368.659	0.762	366.082	0.768	375.388	0.749	373.882	0.752	209.081	1.344
56		267	350.054	0.763	347.607	0.768	357.471	0.747	356.037	0.750	209.081	1.277
57		239	350.054	0.683	347.607	0.688	357.471	0.669	356.037	0.671	209.081	1.143
58		237	298.951	0.793	296.862	0.798	302.606	0.783	301.392	0.786	209.081	1.134
59		249	350.229	0.711	347.781	0.716	351.406	0.709	349.996	0.711	209.081	1.191
60		262	361.913	0.724	359.383	0.729	362.468	0.723	361.014	0.726	209.081	1.253
61		256	361.913	0.707	359.383	0.712	362.468	0.706	361.014	0.709	209.081	1.224
62		213	350.054	0.608	347.607	0.613	357.471	0.596	356.037	0.598	209.081	1.019
63		203	342.806	0.592	340.410	0.596	354.369	0.573	352.947	0.575	209.081	0.971
64		179	333.028	0.537	330.700	0.541	350.121	0.511	348.716	0.513	209.081	0.856
65	Ref. [25]	225	279.819	0.804	276.341	0.814	284.379	0.791	282.461	0.797	182.270	1.234
66		247	324.221	0.762	320.819	0.770	327.549	0.754	325.659	0.758	186.786	1.322
67		224	331.256	0.676	328.392	0.682	334.969	0.669	333.347	0.672	191.195	1.172
68		198	317.816	0.623	315.297	0.628	330.958	0.598	329.474	0.601	192.931	1.026
69		175	308.491	0.567	305.254	0.573	320.832	0.545	318.980	0.549	186.786	0.937
70		192	312.527	0.614	309.598	0.620	325.213	0.590	323.519	0.593	189.444	1.013
71		211	313.859	0.672	311.032	0.678	326.660	0.646	325.018	0.649	190.321	1.109
72		217	258.996	0.838	256.246	0.847	261.638	0.829	260.112	0.834	149.190	1.455
73		260	381.571	0.681	376.929	0.690	385.162	0.675	382.616	0.680	219.454	1.185
74		218	287.862	0.757	284.986	0.765	292.914	0.744	291.297	0.748	187.972	1.160
75		236	304.873	0.774	300.922	0.784	308.180	0.766	306.019	0.771	181.046	1.304
76		240	301.373	0.796	297.981	0.805	305.437	0.786	303.557	0.791	184.992	1.297
77		238	289.639	0.822	285.084	0.835	292.678	0.813	290.218	0.820	174.801	1.362
78		228	262.995	0.867	257.708	0.885	265.831	0.858	263.010	0.867	165.328	1.379
79		268	333.404	0.804	333.510	0.804	340.492	0.787	340.371	0.787	213.957	1.253
80		166	192.618	0.862	183.703	0.904	192.076	0.864	187.482	0.885	117.851	1.409
81	Ref. [26]	68	62.707	1.084	61.936	1.098	60.890	1.117	60.483	1.124	39.589	1.718
82		78	71.024	1.098	70.279	1.110	68.633	1.136	68.237	1.143	40.503	1.926
83		69	60.278	1.145	59.066	1.168	58.132	1.187	57.515	1.200	35.850	1.925
84		62	62.085	0.999	60.336	1.028	59.216	1.047	58.345	1.063	32.325	1.918
85		115	100.703	1.142	99.647	1.154	97.874	1.175	97.309	1.182	63.694	1.806
86		117	112.344	1.041	111.222	1.052	108.589	1.077	107.990	1.083	64.098	1.825
87		118	102.409	1.152	101.026	1.168	99.095	1.191	98.373	1.200	61.317	1.924
88		146	116.200	1.256	114.647	1.273	111.671	1.307	110.865	1.317	61.422	2.377
89	Ref. [27]	324	505.379	0.641	502.707	0.645	527.535	0.614	525.868	0.616	372.114	0.871
90		345	570.477	0.605	566.948	0.609	590.883	0.584	588.750	0.586	367.933	0.938
91		308	402.324	0.766	401.320	0.767	413.591	0.745	412.863	0.746	285.225	1.080
92		330	456.332	0.723	455.241	0.725	465.896	0.708	465.100	0.710	285.596	1.155
93	Ref. [28]	422	469.671	0.899	469.956	0.898	460.054	0.917	459.957	0.917	312.706	1.350
94		438	508.747	0.861	509.057	0.860	475.631	0.921	475.531	0.921	312.706	1.401
95	Ref. [29]	318	240.374	1.323	249.427	1.275	244.941	1.298	249.383	1.275	182.680	1.741
96		343	279.313	1.228	290.383	1.181	282.866	1.213	288.268	1.190	187.061	1.834
97		337	297.537	1.133	309.097	1.090	300.009	1.123	305.625	1.103	185.321	1.818
98		369	315.078	1.171	327.443	1.127	316.822	1.165	322.813	1.143	186.193	1.982
99		286	273.836	1.044	284.688	1.005	286.365	0.999	291.834	0.980	193.768	1.476
100		327	308.900	1.059	321.022	1.019	320.742	1.020	326.807	1.001	192.869	1.695
101		361	282.508	1.278	293.704	1.229	278.320	1.297	283.636	1.273	180.466	2.000
102		402	318.682	1.261	331.188	1.214	311.731	1.290	317.626	1.266	179.628	2.238
103	Ref. [30]	89.5	82.018	1.091	81.548	1.098	74.246	1.205	73.995	1.210	50.264	1.781
104		102.5	95.338	1.075	96.040	1.067	86.871	1.180	87.145	1.176	59.196	1.732
105		129.5	100.530	1.288	101.739	1.273	91.813	1.410	92.316	1.403	62.709	2.065
106		141	108.546	1.299	110.586	1.275	99.465	1.418	100.344	1.405	68.162	2.069
107		98	83.918	1.168	84.515	1.160	76.987	1.273	77.221	1.269	59.015	1.661
108		125.5	105.671	1.188	106.726	1.176	96.021	1.307	96.450	1.301	61.153	2.052
109		138	112.193	1.230	113.390	1.217	101.688	1.357	102.176	1.351	61.676	2.237
110		98	80.041	1.224	80.636	1.215	73.651	1.331	73.886	1.326	59.257	1.654
111		100	84.370	1.185	84.998	1.176	77.408	1.292	77.656	1.288	59.257	1.688
112		117.5	90.542	1.298	91.474	1.285	83.012	1.415	83.395	1.409	61.386	1.914
113		110	90.931	1.210	91.630	1.200	83.104	1.324	83.380	1.319	59.437	1.851
114		88.5	80.246	1.103	80.966	1.093	73.178	1.209	73.468	1.205	49.906	1.773
115		135	130.468	1.035	131.608	1.026	118.962	1.135	119.419	1.130	81.119	1.664
116	Ref. [31]	93.4	92.899	1.005	93.805	0.996	88.612	1.054	88.998	1.049	62.797	1.487
117		102	98.083	1.040	98.434	1.036	92.626	1.101	92.744	1.100	58.167	1.754
118		107.5	106.909	1.006	107.416	1.001	100.611	1.068	100.797	1.066	59.006	1.822
119		113.6	120.172	0.945	121.427	0.936	113.087	1.005	113.618	1.000	63.337	1.794
120		122.2	125.097	0.977	126.404	0.967	117.460	1.040	118.011	1.035	63.337	1.929
121		92.6	111.820	0.828	112.446	0.824	106.007	0.874	106.249	0.872	59.644	1.553

122		111.1	113.457	0.979	113.924	0.975	105.524	1.053	105.687	1.051	58.556	1.897
123		111.3	112.712	0.987	112.993	0.985	104.012	1.070	104.088	1.069	57.380	1.940
124		111.3	118.832	0.937	119.558	0.931	109.126	1.020	109.403	1.017	60.024	1.854
125		82.1	92.210	0.890	91.051	0.902	85.780	0.957	85.196	0.964	47.493	1.729
126		84.9	95.827	0.886	94.940	0.894	89.294	0.951	88.835	0.956	49.521	1.714
127	Ref. [20]	137.5	114.586	1.200	112.282	1.225	113.356	1.213	112.153	1.226	78.696	1.747
128		210.2	210.646	0.998	206.411	1.018	215.109	0.977	212.826	0.988	152.763	1.376
129		297.6	329.862	0.902	323.230	0.921	344.002	0.865	340.352	0.874	247.992	1.200
130		140.8	135.495	1.039	135.694	1.038	134.765	1.045	134.795	1.045	93.115	1.512
131		213.2	257.092	0.829	257.469	0.828	264.220	0.807	264.278	0.807	186.881	1.141
132		290.7	407.857	0.713	408.456	0.712	428.946	0.678	429.040	0.678	308.429	0.943
133		120.8	103.630	1.166	101.019	1.196	101.453	1.191	100.116	1.207	66.460	1.818
134		183.1	211.017	0.868	205.702	0.890	213.462	0.858	210.648	0.869	143.145	1.279
135		231.2	311.939	0.741	304.081	0.760	322.915	0.716	318.658	0.726	220.140	1.050
136		152.3	119.480	1.275	117.618	1.295	118.543	1.285	117.556	1.296	83.357	1.827
137		147.9	135.654	1.090	134.179	1.102	134.373	1.101	133.572	1.107	88.484	1.671
138		158.9	136.471	1.164	134.865	1.178	134.997	1.177	134.132	1.185	87.483	1.816
139	Ref. [32]	290	324.178	0.895	314.787	0.921	312.540	0.928	307.821	0.942	177.088	1.638
140		315	353.594	0.891	343.764	0.916	339.724	0.927	334.797	0.941	179.778	1.752
141		285	286.648	0.994	275.381	1.035	274.882	1.037	269.287	1.058	154.919	1.840
142		310	307.403	1.008	295.242	1.050	293.554	1.056	287.541	1.078	154.402	2.008
143	Ref. [33]	226.5	244.886	0.925	247.103	0.917	241.197	0.939	242.161	0.935	139.531	1.623
144		204	241.979	0.843	244.169	0.835	240.000	0.850	240.959	0.847	139.531	1.462
145		212.8	238.416	0.893	240.574	0.885	238.521	0.892	239.475	0.889	139.531	1.525
146		178.5	234.837	0.760	236.963	0.753	237.022	0.753	237.970	0.750	139.531	1.279
147		249.1	285.097	0.874	289.662	0.860	280.628	0.888	282.721	0.881	152.050	1.638
148		203	281.712	0.721	286.223	0.709	279.235	0.727	281.317	0.722	152.050	1.335
149		222.6	277.564	0.802	282.008	0.789	277.515	0.802	279.584	0.796	152.050	1.464
150		204	273.398	0.746	277.775	0.734	275.771	0.740	277.827	0.734	152.050	1.342
151		263.8	275.174	0.959	277.467	0.951	269.060	0.980	270.041	0.977	138.299	1.907
152		223.6	271.907	0.822	274.173	0.816	267.725	0.835	268.700	0.832	138.299	1.617
153		203	267.903	0.758	270.136	0.751	266.075	0.763	267.044	0.760	138.299	1.468
154		205	263.882	0.777	266.081	0.770	264.404	0.775	265.367	0.773	138.299	1.482
155	Ref. [34]	94.5	100.468	0.941	99.789	0.947	95.296	0.992	94.924	0.996	58.032	1.628
156		112.5	104.140	1.080	103.759	1.084	98.933	1.137	98.701	1.140	60.341	1.864
157		72	93.777	0.768	92.587	0.778	88.684	0.812	88.074	0.817	53.844	1.337
158		108	112.686	0.958	112.322	0.962	106.636	1.013	106.409	1.015	60.664	1.780
159		135	130.957	1.031	131.668	1.025	124.112	1.088	124.385	1.085	67.590	1.997
160		117	116.955	1.000	116.439	1.005	110.298	1.061	109.998	1.064	59.772	1.957
161		99	100.474	0.985	98.085	1.009	93.444	1.059	92.280	1.073	46.761	2.117
162		103.5	100.714	1.028	98.267	1.053	93.598	1.106	92.407	1.120	46.447	2.228
163	Ref. [35]	94.1	103.098	0.913	103.806	0.906	99.556	0.945	99.846	0.942	63.551	1.481
164		111.5	117.570	0.948	118.446	0.941	112.779	0.989	113.140	0.986	64.008	1.742
165		114.6	126.235	0.908	127.154	0.901	120.591	0.950	120.967	0.947	63.877	1.794
166	Ref. [36]	85	116.436	0.730	114.692	0.741	116.345	0.731	115.412	0.736	67.768	1.254
167		91.4	118.637	0.770	116.861	0.782	117.257	0.779	116.316	0.786	67.768	1.349
168		117.5	121.243	0.969	119.427	0.984	118.323	0.993	117.374	1.001	67.768	1.734
169		142.5	124.533	1.144	122.668	1.162	119.651	1.191	118.691	1.201	67.768	2.103
170		148.5	128.907	1.152	126.977	1.170	121.384	1.223	120.410	1.233	67.768	2.191
171		95	94.437	1.006	93.020	1.021	93.448	1.017	92.697	1.025	67.743	1.402
172		102.5	107.337	0.955	105.756	0.969	105.494	0.972	104.661	0.979	67.985	1.508
173		113	116.102	0.973	114.447	0.987	113.674	0.994	112.803	1.002	68.393	1.652
174		110	124.457	0.884	122.444	0.898	121.116	0.908	120.071	0.916	66.744	1.648
175		99.35	124.485	0.798	122.125	0.814	120.751	0.823	119.540	0.831	64.421	1.542
176		72	77.280	0.932	73.199	0.984	73.957	0.974	71.940	1.001	41.536	1.733
177		82	93.166	0.880	89.693	0.914	89.887	0.912	88.151	0.930	50.895	1.611
178		96.5	94.670	1.019	91.267	1.057	91.402	1.056	89.698	1.076	51.789	1.863
179		108.4	101.483	1.068	98.429	1.101	98.277	1.103	96.737	1.121	55.853	1.941
180		130	132.826	0.979	131.879	0.986	130.143	0.999	129.612	1.003	74.833	1.737

Ravg. =	0.999	Ravg. =	0.998	Ravg. =	1.004	Ravg. =	1.004	Ravg. =	1.652
Correl.=	0.836	Correl.=	0.846	Correl.=	0.834	Correl.=	0.839	Correl.=	0.813
St Dev.=	0.282	St Dev.=	0.270	St Dev.=	0.292	St Dev.=	0.288	St Dev.=	0.511
Var.=	0.080	Var.=	0.073	Var.=	0.086	Var.=	0.083	Var.=	0.261
Max. R=	2.029	Max. R=	1.954	Max. R=	1.993	Max. R=	1.957	Max. R=	3.752
Min. R=	0.521	Min. R=	0.518	Min. R=	0.451	Min. R=	0.450	Min. R=	0.672

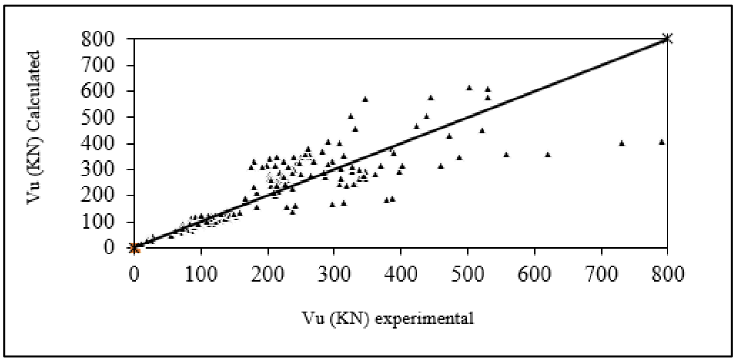

Figure 2: Experimental and calculated results of ultimate punching shear (Vu) form equation (77)

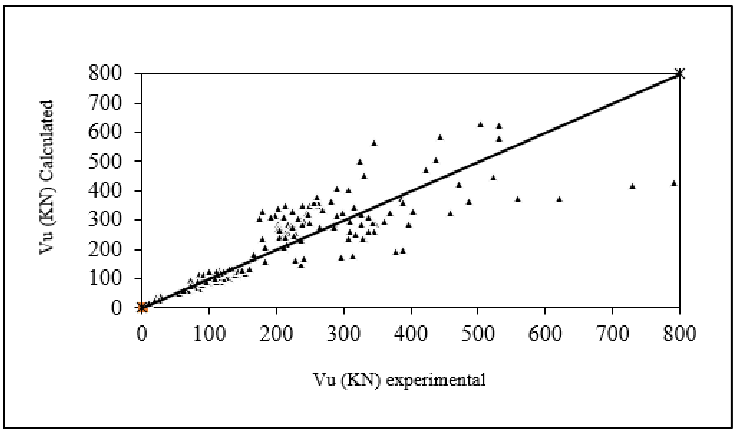

Figure 3: Experimental and calculated results of ultimate punching shear (Vu) form equation (78)

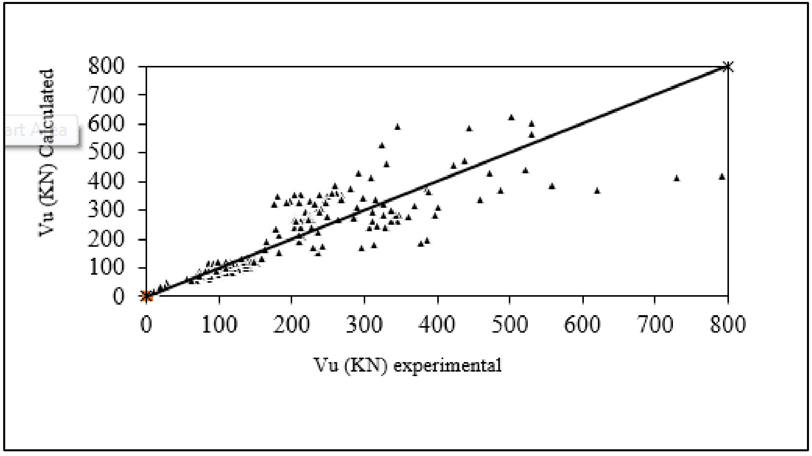

Figure 4: Experimental and calculated results of ultimate punching shear (Vu) form equation (80)

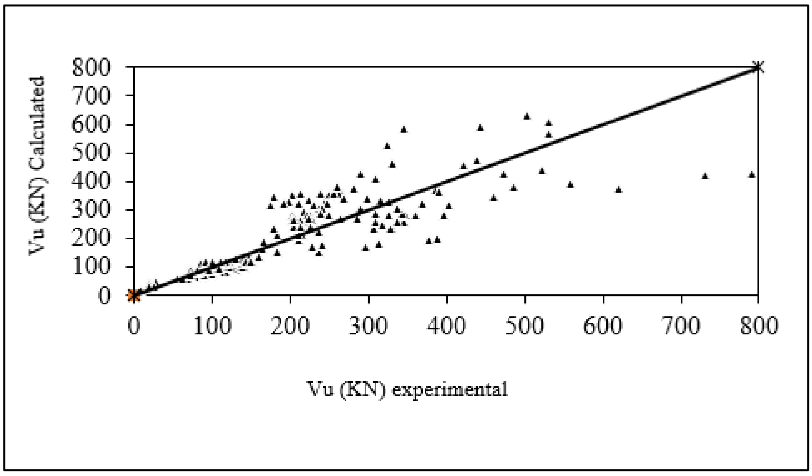

Figure 5: Experimental and calculated results of ultimate punching shear (Vu) form equation (81)

Table 3: Predicted results for cracking punching shear of concrete slabs with steel fiber

$$V_{cr} = 1.95 x\, 10^{-4}\,(f_c^{'})^{0.5}\,(\rho f_y)^{-0.05}\,F^{0.5}\,b_o d \tag{1}$$

$$V_{cr} = 1.9 x\, 10^{-4}\,(f_c^{'})^{0.5}\,(\rho)^{-0.35}\,F^{0.5}\,b_o d \tag{2}$$

$$V_{cr} = 0.333 x\, 10^{-3}\,(f_c^{'})^{0.5}\,b_o d \quad \text{(ACI – Code)} \tag{3}$$

$$V_{cr} = 0.167 x\, 10^{-3}\,(f_c^{'})^{0.5}\,b_o d \quad \text{(ACI – Code)} \tag{4}$$

			Eq.(1) 82		Eq.(2) 83		Eq.(3) 84		Eq.(4) 85	
No.	Reference	$V_{cr\,\exp}$ (KN)	$V_{cr\,cal}$ (KN)	$R = \frac{V_{cr\,\exp}}{V_{cr\,cal}}$	$V_{cr\,cal}$ (KN)	$R = \frac{V_{cr\,\exp}}{V_{cr\,cal}}$	$V_{cr\,cal}$ (KN)	$R = \frac{V_{cr\,\exp}}{V_{cr\,cal}}$	$V_{cr\,cal}$ (KN)	$R = \frac{V_{cr\,\exp}}{V_{cr\,cal}}$
1	Ref. [21]	30	52.622	0.570	62.896	0.477	168.661	0.178	84.331	0.356
2		40	65.933	0.607	78.806	0.508	172.546	0.232	86.273	0.464
3		45	79.200	0.568	94.663	0.475	179.497	0.251	89.748	0.501
4		35	52.622	0.665	62.896	0.556	168.661	0.208	84.331	0.415
5		40	65.933	0.607	78.806	0.508	172.546	0.232	86.273	0.464
6		45	79.200	0.568	94.663	0.475	179.497	0.251	89.748	0.501
7		46	52.622	0.874	62.896	0.731	168.661	0.273	84.331	0.545
8		50	65.933	0.758	78.806	0.634	172.546	0.290	86.273	0.580
9		55	79.200	0.694	94.663	0.581	179.497	0.306	89.748	0.613
10	Ref. [13]	6.7	7.787	0.860	6.437	1.041	24.064	0.278	12.032	0.557
11		5.5	7.753	0.709	6.409	0.858	23.959	0.230	11.980	0.459
12		5.3	7.871	0.673	6.506	0.815	24.323	0.218	12.161	0.436
13		6.6	9.236	0.715	7.635	0.864	22.475	0.294	11.237	0.587
14		5.1	13.127	0.389	10.851	0.470	22.586	0.226	11.293	0.452
15		4.5	15.878	0.283	13.125	0.343	22.307	0.202	11.153	0.403
16		9.1	20.118	0.452	16.630	0.547	24.477	0.372	12.238	0.744
17		3.1	4.438	0.699	3.246	0.955	13.996	0.221	6.998	0.443
18		15.5	14.482	1.070	14.102	1.099	43.547	0.356	21.773	0.712
19		23.9	19.512	1.225	20.206	1.183	58.077	0.412	29.039	0.823
20		5.5	7.025	0.783	5.807	0.947	21.709	0.253	10.854	0.507
21		7	8.918	0.785	7.372	0.950	27.560	0.254	13.780	0.508
22		6.2	7.371	0.841	6.093	1.018	22.780	0.272	11.390	0.544
23		5.3	7.227	0.733	5.974	0.887	22.335	0.237	11.167	0.475
24	Ref. [30]	10	15.096	0.662	14.324	0.698	50.264	0.199	25.132	0.398
25		14.75	17.779	0.830	16.870	0.874	59.196	0.249	29.598	0.498
26		16	18.834	0.850	17.871	0.895	62.709	0.255	31.354	0.510
27		20	20.472	0.977	19.425	1.030	68.162	0.293	34.081	0.587
28		14	12.533	1.117	11.892	1.177	59.015	0.237	29.508	0.474
29		17	22.495	0.756	21.344	0.796	61.153	0.278	30.577	0.556
30		19.5	26.217	0.744	24.875	0.784	61.676	0.316	30.838	0.632
31		13	10.920	1.190	10.362	1.255	59.257	0.219	29.628	0.439
32		17	12.641	1.345	11.995	1.417	59.257	0.287	29.628	0.574
33		19	14.558	1.305	13.813	1.375	61.386	0.310	30.693	0.619
34		18	15.491	1.162	14.698	1.225	59.437	0.303	29.718	0.606
35		14.5	14.989	0.967	14.222	1.020	49.906	0.291	24.953	0.581
36		15	24.364	0.616	23.117	0.649	81.119	0.185	40.560	0.370
37	Ref. [36]	35	37.700	0.928	44.889	0.780	67.768	0.516	33.884	1.033
38		45	37.406	1.203	42.502	1.059	67.768	0.664	33.884	1.328
39		55	37.069	1.484	39.893	1.379	67.768	0.812	33.884	1.623
40		60	36.658	1.637	36.896	1.626	67.768	0.885	33.884	1.771
41		70	36.135	1.937	33.362	2.098	67.768	1.033	33.884	2.066
42		45	18.528	2.429	19.939	2.257	67.743	0.664	33.872	1.329
43		50	26.296	1.901	28.299	1.767	67.985	0.735	33.992	1.471
44		55	32.399	1.698	34.867	1.577	68.393	0.804	34.197	1.608
45		57.5	40.819	1.409	43.928	1.309	66.744	0.861	33.372	1.723
46		55	43.159	1.274	46.446	1.184	64.421	0.854	32.211	1.708
47		25	22.720	1.100	24.451	1.022	41.536	0.602	20.768	1.204
48		35	27.840	1.257	29.961	1.168	50.895	0.688	25.448	1.375
49		40	28.329	1.412	30.487	1.312	51.789	0.772	25.894	1.545
50		47.5	30.552	1.555	32.879	1.445	55.853	0.850	27.926	1.701
51		55	40.935	1.344	44.053	1.249	74.833	0.735	37.417	1.470

Ravg. =	1.004	Ravg. =	1.007	Ravg. =	0.411	Ravg. =	0.821
Correl.=	0.728	Correl.=	0.703	Correl.=	0.561	Correl.=	0.561
St Dev.=	0.436	St Dev.=	0.413	St Dev.=	0.245	St Dev.=	0.489
Var.=	0.190	Var.=	0.171	Var.=	0.060	Var.=	0.239
Max. R=	2.429	Max. R=	2.257	Max. R=	1.033	Max. R=	2.066
Min. R=	0.283	Min. R=	0.343	Min. R=	0.178	Min. R=	0.356

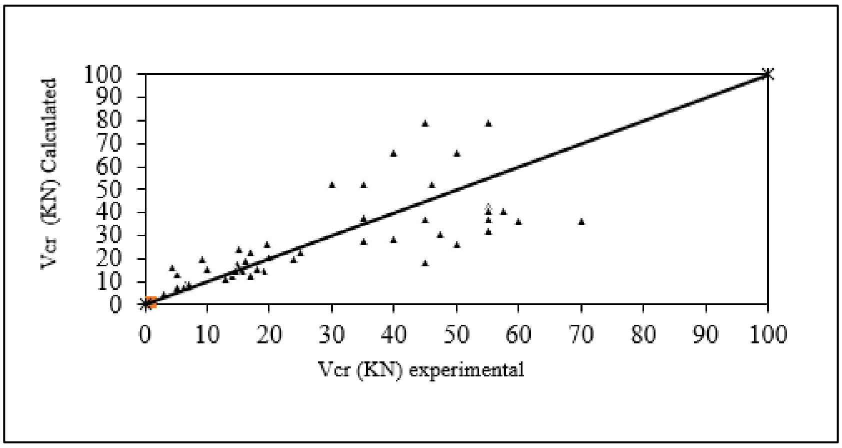

Figure 6: Experimental and calculated results of cracking punching shear (Vcr) form equation (82)

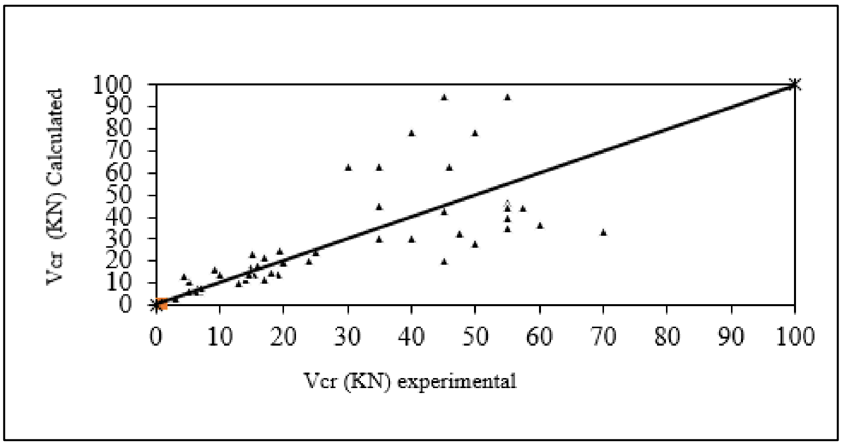

Figure 7: Experimental and calculated results of cracking punching shear (Vcr) form equation (83)

4. Conclusions

1. Theoretical equations are proposed to predict the cracking and ultimate punching shear of slabs with steel fiber taking into account the effect of the steel fiber properties, (volume content, aspect ratio and type of steel fiber), compressive strength and tensile strength of the concrete, conventional flexural steel reinforcement Index ratio and yield strength of reinforcing bar, critical section perimeter and effective depth of the slab.

2. Different mathematical models are tested (linear equation, power, exponential & reciprocal) forms to find the relationship between the shear strength of the slab and the considering variables mentioned above.

3. Multi-linear and multi non-linear (multi-power) equations are proposed to find the best relationship between the variables under consideration and the shear strength of the slabs.

4. Regression analysis is used to find the values of the coefficients of the proposed equations by using the database of experimental results found in literature. The theoretical results obtained from these proposed equations showed good agreement with the experimental data.

References

[1] J. Moe, “Shearing strength of reinforced slabs on footings under concentrated loads,” Portland Cement Association, Research and Development Laboratories, April 1961.

[2] ACI – Committee 544; “State of Art Report on Fiber Reinforced Concrete,” sp-81 ACI Publication, Detroit, Mechigan, pp. 411-452, 1984.

[3] J. K. Ibrahim, “Punching shear strength of steel fiber reinforced concrete slabs,” M.Sc. Thesis, University of Baghdad, Iraq, 1984.

[4] D. Yitizaki, “Punching strength of reinforced concrete slabs,” ACI Journal, pp. 527-540, May, 1966.

[5] M. Herzog, “New evaluation of earlier punching shear test,” Concrete Research, December, pp. 448-450, 1970.

[6] M. A. Al. Ani, “Influence of steel fiber on punching shear strength of Reinforced concrete slabs,” M.Sc. Thesis, University of Baghdad, Iraq, 1985.

[7] N. K. Oukaili and T. S. Salman, “Punching shear strength of reinforced concrete flat plates with openings,” Journal of Engineering, Vol.20; No.1, pp. 1-20, January 2014.

[8] A. H. N. Al-Mamoori, “Investigation the punching shear behavior of reinforced concrete slab-column connection using carbon fiber reinforced polymers,” Al-Qadisiya Journal for Engineering Science, Vol.8, No.1, pp. 38-58, 2015.

[9] K. F. Sarsam and H. F. Hassan, “Punching shear failure characteristics of flat slabs using reactive and modified powder concrete with steel fiber,” Journal of Engineering and Development, Vol.17, No.15, pp. 124-140, Nov., 2013.

[10] H. Ju, N.R. Cheon, D. H. Lee, J. V. Oh, J. H. Hwang, and K. S. Kim, “Consideration on punching shear strength of steel fiber reinforced concrete slabs,” Advanced in Mechanical Engineering; Vol.7, No.5, pp. 1-12, 2015.

[11] B. Mu and C. Meyer, “Bending and punching shear strength of Fiber-Reinforced Glass concrete slabs,” Nov. 2002.

[12] J. B. Hanai and K. M. A. Holanda, “Similarities between punching and shear strength of steel fiber reinforced concrete (SFRC) slabs and beams,” Ibracon Structures and Materials Journal, Vol.1 No.1, pp. 1-16, March, 2008.

[13] V. B. Jatale and L. G. Kalurkar, “Punching shear high strength steel fiber reinforced concrete slabs,” International Journal of Engineering Research and Applications; Vol.3, Issue 6, pp. 269-275, Nov-Dec., 2013.

[14] T. A. Q. Al-Shaikhli, “Effect of steel fiber on punching shear strength on non-rectangular reactive powder concrete slabs,” International Journal of Structural and Civil Engineering Research, Vol.5, No.2, pp.130-135, May 2016.

[15] K. K. Choi, M. M. R. Taha, H. H. Park and A. K. Maji, “Punching shear strength of interior concrete slab-column connections reinforced with steel fibers,” Cement and Concrete Composites, Vol.29, pp. 409-420, 2007.

[16] L. F. Maya, M. F. Ruiz, A. Muttoni and S. J. Foster, “punching shear strength of steel fiber reinforced concrete slabs,” Engineering Structures, Vol.40, pp.83-94, 2012.

[17] K. S. Rajab “Study punching shear of steel fiber reinforced self compacting concrete slabs by Nonlinear analysis,” International Journal of Civil, Environmental, Structural, Construction and Architectural Engineering, Vol.17, No.9, pp. 624-635, 2013.

[18] M. Y. Cheng and G. J. P. Montensions, “Evaluation of steel fiber reinforcement for punching shear resistance in slab-column connections part I: Monotonically Increased load,” ACI-Structural Journal, Vol.107, No.1, Jan-Feb., 2010.

[19] S. R. Magdum and S. A. Veerabhadrannavap, “Finit Element Analysis of Conventional Reinforced concrete and steel fiber Reinforced concrete in slab-column joint,” International Journal of Research in Engineering and Technology, Vol.5, Issue 10, pp. 74-81, Oct., 2016.

[20] H. Higashiyama, A. Ota and M. Mizukoshi, “Design equation for punching shear capacity of SFRC slabs,” International Journal of Concrete Structures and Materials, Vol.5, No.1, pp. 35-42, June, 2011.

[21] L. N. Minh, M. Rovnak, J. J. Quoc and K. N. Kin, “Punching shear Resistance of steel fiber reinforced concrete flat slabs,” Procedia Engineering, Research gate, pp1-8, Dec. 2011.

[22] E. Mondo, “Shear Capacity of steel fiber reinforced concrete beams without conventional shear reinforcement,” M.Sc. Thesis, Department of Civil and Architectural Engineering, Division of structured Design and Bridges, Royal Institute of Technology, pp.113, Stockholm, Sweden, 2011.

[23] H. A. A. Al-Quraishi, “Punching shear behavior of UHPC flat slabs,” Ph.D. Thesis; Faculty of Civil and Environmental Engineering, University of Kassel, Kassel, Germany, 2014.

[24] R. N. Swamy and Ali SAR, “Punching shear behavior of reinforced slab-column connections made with steel fiber concrete,” ACI-Journal, Vol. 79, No.6, pp. 392-406, 1982.

[25] D.D. Theodorakopoulos and N. Swamy, “Contribution of steel fibers to the strength characteristics of light weight concrete slab-column connections falling in punching shear,” ACI- structural Journal, Vol. 90, No.4, pp. 342-355, 1993.

[26] M. It. Harajli, D. Maalouf and H. Khatib, “Effect of fibers on the punching shear strength of slab-column connections,” Cement and Concrete Composite, vol. 17, pp.161-170, 1995.

[27] S. D. B. Alexander and S. H. Simmonds, “Punching shear tests of concrete slab-column joints containing fiber reinforcement,” ACI- structural Journal; Vol. 89, No.4, pp. 425-432, 1992.

[28] P .J. McHarg, W. D. Cook, D. Mitchell and Yoon Young-Soo, “Benefits of concentrated slab reinforcement and steel fibers on performance of slab-column connections,” ACI-Structural Journal, Vol. 97, No.2, pp. 225-234, 2000.

[29] R. Suter and L. Moreillan, “Punching shear strength of high performance fiber reinforced concrete slabs,” 3rd FIB international congress, Washington, USA, 2010.

[30] A. Yassen “Punching shear strength of steel fiber high strength reinforced concrete slabs,” Msc. Thesis, University of Salahaddin, College of Engineering, Erbil, Iraq; Dec. 2006.

[31] R. Naranayan and I. Y. S. Darwish, “Punching shear tests on steel-fiber-reinforced micro-concrete slabs,” Mag Concrete Research, Vol. 39, No. 138, pp. 42-50, 1987.

[32] X. Wang, W. Tian, Z. Huang, M. Zhou and X. Zhao, “Analysis on Punching shear behavior of the raft slab Reinforced with steel fibers,” Advanced concrete and structure; vol. 400; 2009, pp. 335-340.

[33] M. Hirasawa, K. Ito and K. Takagi, “Punching shear strength at steel fiber reinforced concrete slabs,” Proceedings of the 37th Annual Conference of the JSCE, Vol. 124, pp. 247-258, 1982.

[34] A. M. Shaahan and H. Gesund, “Punching shear strength of steel fiber reinforced concrete flat slabs,” ACI structural Journal, Vol. 91, No.3, pp.406-414, 1994.

[35] M. Wakabayashi and T. Maruyama, “A study on punching shear Resistance of reinforced concrete slabs by using steel fiber,” Proceedings of the 57th Annual conference of the JSCE, Vol. 131, 2002.

[36] G. S. Basman, “Strength of reinforced concrete slabs for punching shear stress,” M.Sc. Thesis, University of Salahaddin, College of Engineering. Erbil, Iraq, 1989.

[37] D. J. Hannant, “Fiber cements and fiber concretes,” John Wiley & Sons, publication, New York, USA, 1978.

[38] R. N. Swamy, R. S. Mangar and K. C. U. S. Roa, K. C. U. S. “The Mechanics of fiber reinforcement of cement materials,” American Concrete Institute, Detroit, Sp-44, pp. 1-28, 1974.

[39] R. Narayanan and S. S. K. Palanjlain, “A space truss model for fiber concrete beams in torsion,” The structural Engineer, vol. 63B, No.1, March, pp. 4-18, 1985.

[40] R. Narayanan and A. S. K. Palanjain, “Effect of fiber addition on concrete strength,” Indian Concrete Journal, Vol. 58, No. 4, pp.100-103, April 1984.

[41] ACI 318-14; “Building Code Requirements for structural concrete”; American Concrete Institute; USA; pp. 524, 2014.